Math 101 Chapter 10 Review

#1. $3\sqrt{49} = 3 \cdot 7$
$= 21$

SDWK
49
7 7
$49 = 7^2$

#2. $\sqrt{9+16} = \sqrt{25}$
$= 5$

SDWK
25
5 5
$25 = 5^2$

#3. $\sqrt[3]{-27} = -3$

SDWK
27
3 9
3 3
$-27 = (-3)(-3)(-3)$

#4. $6\sqrt[4]{16x^4} = 6\sqrt[4]{2^4 \cdot x^4}$
$= 6 \cdot 2 \cdot |x|$
$= 12|x|$

SDWK
16
2 8
2 4
2 2
$16 = 2^4$
$\sqrt[4]{x^4} = |x|$

↖ Even roots need absolute values

#5. $\sqrt{(2x-1)^2} = |2x-1|$

ODD Roots DONT Need absolute values

#6 $\sqrt[3]{(4x+2)^3} = 4x+2$

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#7. $f(x)=\sqrt{9-4x}$

Find the domain! ← You can't take the square root of a negative

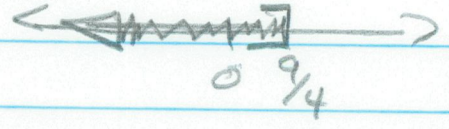
Solve: $9-4x \geq 0$

$4x+9-4x \geq 4x+0$

$9 \geq 4x$

$\frac{1}{4} \cdot \frac{9}{1} \geq \frac{1}{4} \cdot 4x$

$\frac{9}{4} \geq x$



The domain of $f(x)=\sqrt{9-4x}$ is $\{x \mid \frac{9}{4} \geq x\} = (-\infty, \frac{9}{4}]$.

#8. $f(x)=\sqrt{9-4x}$

$f(2)=\sqrt{9-4(2)}$

$f(2)=\sqrt{9-8}$

$f(2)=\sqrt{1}$

$f(2)=1$

#9 $f(x)=\sqrt{9-4x}$

$f(x)=3$

$3=\sqrt{9-4x}$

$(3)^2=(\sqrt{9-4x})^2$

$9=9-4x$

$-9+9=-9+9-4x$

$\frac{0}{-4}=\frac{-4x}{-4}$

$0=x$

check

$3=\sqrt{9-4(0)}$

$3=\sqrt{9-0}$

$3=\sqrt{9}$

$3=3$

TRUE!

The solution set is $\{0\}$.

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Math 101 Chapter 10 Review

#10 $$\frac{\sqrt[4]{x}}{\sqrt[5]{x^2}} = \frac{x^{\frac{1}{4}}}{x^{2/5}}$$

$$= x^{\frac{1}{4}-\frac{2}{5}}$$

$$= x^{\frac{1}{4}\cdot\frac{5}{5}-\frac{2}{5}\cdot\frac{4}{4}}$$

$$= x^{\frac{5}{20}-\frac{8}{20}}$$

$$= x^{-\frac{3}{20}}$$

$$= \frac{1}{x^{3/20}}$$

$$= \frac{1}{\sqrt[20]{x^3}}$$

#11. $$\sqrt[5]{\sqrt[3]{2x}} = \left((2x)^{\frac{1}{3}}\right)^{1/5}$$

$$= (2x)^{\frac{1}{3}\cdot\frac{1}{5}}$$

$$= (2x)^{\frac{1}{15}}$$

$$= \sqrt[15]{2x}$$

#12. $$\sqrt{75x^3y^4} = \sqrt{5^2x^2y^4}\cdot\sqrt{3x}$$

$$= 5xy^2\sqrt{3x}$$

SDWK

$75 = 3 \cdot 5^2$

75

3 25

5 5

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#13. $\sqrt[5]{32x^{11}y^9} = \sqrt[5]{2^5 x^{10} y^5} \cdot \sqrt[5]{x y^4}$

$= 2x^2 y \sqrt[5]{x y^4}$

SDWK

32 → 4, 8 → 2, 2, 2, 4 → 2, 2

$32 = 2^5$

#14. $\sqrt{5xy} \cdot \sqrt{10xy^2} = \sqrt{50x^3y^3}$

$= \sqrt{5^2 x^2 y^2} \cdot \sqrt{2xy}$

$= 5xy\sqrt{2xy}$

SDWK

50 → 5, 10 → 5, 2

$50 = 5^2 \cdot 2$

#15. $\sqrt[3]{6x^7y} \cdot \sqrt[3]{9x^4y^{12}} = \sqrt[3]{54x^{11}y^{13}}$

$= \sqrt[3]{3^3 x^9 y^{12}} \cdot \sqrt[3]{2x^2y}$

$= 3x^3y^4\sqrt[3]{2x^2y}$

SDWK

54 → 2, 27 → 3, 9 → 3, 3

$54 = 3^3 \cdot 2$

#16. $3\sqrt{32x} - 2\sqrt{18x}$

$= 3 \cdot \sqrt{2^4} \cdot \sqrt{2x} - 2\sqrt{3^2} \cdot \sqrt{2x}$

$= 3 \cdot 2^2 \cdot \sqrt{2x} - 2 \cdot 3 \cdot \sqrt{2x}$

$= 3 \cdot 4 \cdot \sqrt{2x} - 6\sqrt{2x}$

$= 12\sqrt{2x} - 6\sqrt{2x}$

$= 6\sqrt{2x}$

SDWK

32 → 4, 8 → 2, 2, 2, 4 → 2, 2

$32 = 2^4 \cdot 2$

18 → 2, 9 → 3, 3

$18 = 3^2 \cdot 2$

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#17. $\sqrt[3]{\frac{50x^8}{27y^{12}}} = \frac{\sqrt[3]{50x^8}}{\sqrt[3]{27y^{12}}}$

$= \frac{\sqrt[3]{x^6 \cdot 2 \cdot 5^2 x^2}}{\sqrt[3]{3^3 y^{12}}}$

$= \frac{x^2 \sqrt[3]{50x^2}}{3y^4}$

SDWK

50 $\quad 50 = 2 \cdot 5^2$

2 25

5 5

27 $\quad 27 = 3^3$

3 9

3 3

#18. $\sqrt{5}(2x + \sqrt{7})$

$= (2x)(\sqrt{5}) + (\sqrt{7})(\sqrt{5})$

$= 2x\sqrt{5} + \sqrt{35}$

#19. $(6 - \sqrt{11})(4 + \sqrt{2})$

$= (6)(4) + (6)(\sqrt{2}) + (-\sqrt{11})(4) + (-\sqrt{11})(\sqrt{2})$

$= 24 + 6\sqrt{2} - 4\sqrt{11} - \sqrt{22}$

#20. $\frac{7\sqrt{2x}}{\sqrt{y}} = \frac{7\sqrt{2x}}{\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}}$

$= \frac{7\sqrt{2xy}}{\sqrt{y^2}}$

$= \frac{7\sqrt{2xy}}{y}$

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#21. $\dfrac{2}{\sqrt{x}-3} = \left(\dfrac{2}{\sqrt{x}-3}\right)\cdot\left(\dfrac{\sqrt{x}+3}{\sqrt{x}+3}\right)$

$$= \frac{2\cdot(\sqrt{x}+3)}{(\sqrt{x})^2-(3)^2}$$

$$= \frac{2\sqrt{x}+6}{x-9}$$

#22. $\dfrac{3+\sqrt{x}}{4-\sqrt{x}} = \left(\dfrac{3+\sqrt{x}}{4-\sqrt{x}}\right)\cdot\left(\dfrac{4+\sqrt{x}}{4+\sqrt{x}}\right)$

$$= \frac{(3+\sqrt{x})(4+\sqrt{x})}{(4)^2-(\sqrt{x})^2}$$

$$= \frac{(3)(4)+(3)(\sqrt{x})+(\sqrt{x})(4)+(\sqrt{x})(\sqrt{x})}{16-x}$$

$$= \frac{12+3\sqrt{x}+4\sqrt{x}+x}{16-x}$$

$$= \frac{12+7\sqrt{x}+x}{16-x}$$

#23. $\dfrac{2x}{\sqrt[3]{3x}} = \dfrac{2x}{\sqrt[3]{3x}}\cdot\dfrac{\sqrt[3]{3^2x^2}}{\sqrt[3]{3^2x^2}}$

$$= \frac{2x\cdot\sqrt[3]{3^2x^2}}{\sqrt[3]{3^3x^3}}$$

$$= \frac{2\cdot x\cdot\sqrt[3]{9x^2}}{3x}$$

$$= \frac{2\sqrt[3]{9x^2}}{3}$$

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#24. $\dfrac{5x^2}{\sqrt[4]{3x^2y^3}} = \dfrac{5x^2}{\sqrt[4]{3x^2y^3}} \cdot \dfrac{\sqrt[4]{3^3x^2y}}{\sqrt[4]{3^3x^2y}}$

$= \dfrac{5x^2 \cdot \sqrt[4]{3^3x^2y}}{\sqrt[4]{3^4x^4y^4}}$

$= \dfrac{5 \cdot x^2 \cdot \sqrt[4]{27x^2y}}{3xy}$

$= \dfrac{5x\sqrt[4]{27x^2y}}{3y}$

SDWK

$3^3 = 3 \cdot 3 \cdot 3$

$= 9 \cdot 3$

$= 27$

#25. $\sqrt{2x-3} = 7$

$(\sqrt{2x-3})^2 = (7)^2$

$2x - 3 = 49$

$2x - 3 + 3 = 49 + 3$

$2x = 52$

$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot \frac{52}{1}$

$x = 26$

check:

$\sqrt{2(26)-3} = 7$

$\sqrt{52-3} = 7$

$\sqrt{49} = 7$

$7 = 7$

TRUE

$\{26\}$

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#26. $x = \sqrt{3x+7} - 3$

$x + 3 = \sqrt{3x+7} - 3 + 3$

$x + 3 = \sqrt{3x+7}$

$(x+3)^2 = (\sqrt{3x+7})^2$

$(x+3)(x+3) = 3x+7$

$x^2 + 3x + 3x + 9 = 3x + 7$

$x^2 + 6x + 9 = 3x + 7$

$-3x - 7 + x^2 + 6x + 9 = -(3x+7) + 3x + 7$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

Either

$x + 2 = 0$, or $x + 1 = 0$

$-2 + x + 2 = -2 + 0$ $\quad -1 + x + 1 = -1 + 0$

$x = -2$ $\quad x = -1$

$\{-2, -1\}$

Check

$(-2) = \sqrt{3(-2)+7} - 3$

$-2 = \sqrt{-6+7} - 3$

$-2 = \sqrt{1} - 3$

$-2 = 1 - 3$

$-2 = -2$

TRUE!

$(-1) = \sqrt{3(-1)+7} - 3$

$-1 = \sqrt{-3+7} - 3$

$-1 = \sqrt{4} - 3$

$-1 = 2 - 3$

$-1 = -1$

TRUE!

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#27. $\sqrt{x-8} = \sqrt{x}-2$

$(\sqrt{x-8})^2 = (\sqrt{x}-2)^2$

$x-8 = (\sqrt{x}-2)(\sqrt{x}-2)$

$x-8 = x-2\sqrt{x}-2\sqrt{x}+4$

$x-8 = x+4-4\sqrt{x}$

$-x-4+x-8 = -(x+4)+x+4-4\sqrt{x}$

$-12 = -4\sqrt{x}$

$\frac{-12}{-4} = \frac{-4\sqrt{x}}{-4}$

$3 = \sqrt{x}$

$(3)^2 = (\sqrt{x})^2$

$9 = x$

$\{9\}$

check

$\sqrt{(9)-8} = \sqrt{(9)}-2$

$\sqrt{1} = 3-2$

$1 = 1$

TRUE!

#28. $\sqrt[3]{4x-5} = 5$

$(\sqrt[3]{4x-5})^3 = (5)^3$

$4x-5 = 125$

$5+4x-5 = 5+125$

$4x = 130$

$\frac{4x}{4} = \frac{130}{4}$

$x = 32.5$

$\{32.5\}$

check

$\sqrt[3]{4(32.5)-5} = 5$

$\sqrt[3]{130-5} = 5$

$\sqrt[3]{125} = 5$

$5 = 5$

TRUE!

spuk

$5^3 = 5 \cdot 5 \cdot 5$

$= 25 \cdot 5$

$= 125$

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#29, $\sqrt{2x+3} + \sqrt{x+1} = 1$

$-\sqrt{x+1} + \sqrt{2x+3} + \sqrt{x+1} = -\sqrt{x+1} + 1$

$\sqrt{2x+3} = -\sqrt{x+1} + 1$

$(\sqrt{2x+3})^2 = (-\sqrt{x+1} + 1)^2$

$2x+3 = (-\sqrt{x+1} + 1)(-\sqrt{x+1} + 1)$

$2x+3 = (-\sqrt{x+1})(-\sqrt{x+1}) + (-\sqrt{x+1})(1) + (-\sqrt{x+1})(1) + (1)(1)$

$2x+3 = x+1 - \sqrt{x+1} - \sqrt{x+1} + 1$

$2x+3 = x+2 - 2\sqrt{x+1}$

$-x-2+2x+3 = -(x+2) + x + 2 - 2\sqrt{x+1}$

$x+1 = -2\sqrt{x+1}$

$(x+1)^2 = [2\sqrt{x+1}]^2$

$(x+1)(x+1) = (-2)^2 \cdot (\sqrt{x+1})^2$

$x^2 + x + x + 1 = 4 \cdot (x+1)$

$x^2 + 2x + 1 = 4x + 4$

$-4x - 4 + x^2 + 2x + 1 = -(4x+4) + 4x + 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

Either

$x - 3 = 0$, or $x + 1 = 0$

$3 + x - 3 = 3 + 0$ | $-1 + x + 1 = -1 + 0$

$x = 3$ | $x = -1$

$\{-1, 3\}$

check

$\sqrt{2(3)+3} + \sqrt{(3)+1} = 1$

$\sqrt{6+3} + \sqrt{3+1} = 1$

$\sqrt{9} + \sqrt{4} = 1$

$3 + 2 = 1$

$5 = 1$

False!

$\sqrt{2(-1)+3} + \sqrt{(-1)+1} = 1$

$\sqrt{-2+3} + \sqrt{-1+1} = 1$

$\sqrt{1} + \sqrt{0} = 1$

$1 = 1$

TRUE!

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#30. $\sqrt{-64} = \sqrt{64} \cdot \sqrt{-1}$
$= \sqrt{8^2} \cdot i$
$= 8i$

SDWK
$64 = 8^2$

#31. $4\sqrt{-75} = 4 \cdot \sqrt{75} \cdot \sqrt{-1}$
$= 4\sqrt{5^2} \cdot \sqrt{3} \cdot i$
$= 4 \cdot 5 \cdot \sqrt{3} \cdot i$
$= 20 \cdot \sqrt{3} \cdot i$
$= \underline{20i\sqrt{3}}$ or $20\sqrt{3}\,i$

SDWK
75 $75 = 5^2 \cdot 3$
3 25
5 5

#32. $\sqrt{-64} \cdot \sqrt{-9} = [\sqrt{64} \cdot \sqrt{-1}] \cdot [\sqrt{9} \cdot \sqrt{-1}]$
$= [\sqrt{8^2} \cdot i] \cdot [\sqrt{3^2} \cdot i]$
$= [8 \cdot i] \cdot [3 \cdot i]$
$= 24\,i^2$
$= 24(-1)$
$= -24$

SDWK
$64 = 8^2$
$9 = 3^2$
$i^2 = -1$

#33. $(7 - 2i)(6 + 5i)$
$= (7)(6) + (7)(5i) + (-2i)(6) + (-2i)(5i)$
$= 42 + 35i - 12i - 10i^2$
$= 42 + 23i - 10(-1)$
$= 42 + 23i + 10$
$= 52 + 23i$

SDWK
$i^2 = -1$

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#34. $(7-2i)+(6+5i)$
$= (7+6)+(-2i+5i)$
$= 13+3i$

#35. $(7-2i)-(6+5i)$
$= 7-2i-6-5i$
$= (7-6)+(-2i-5i)$
$= 1+(-7i)$
$= 1-7i$

#36. $\frac{11}{1-3i} = \left(\frac{11}{1-3i}\right)\left(\frac{1+3i}{1+3i}\right)$
$= \frac{11\cdot(1+3i)}{(1)^2-(3i)^2}$
$= \frac{11+33i}{1-(-9)}$
$= \frac{11+33i}{1+9}$
$= \frac{11+33i}{10}$ or $\frac{11}{10}+\frac{33}{10}i$

SPWK

$(3i)^2$
$= 3^2 \cdot i^2$
$= 9\cdot(-1)$
$= -9$

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#37. $\dfrac{6-4i}{5+2i} = \left[\dfrac{6-4i}{5+2i}\right]\left[\dfrac{5-2i}{5-2i}\right]$

$= \dfrac{(6)(5)+(6)(-2i)+(-4i)(5)+(-4i)(-2i)}{(5)^2-(2i)^2}$

$= \dfrac{30-12i-20i+8i^2}{25-(-4)}$

$= \dfrac{30-32i+8(-1)}{25+4}$

$= \dfrac{30-32i-8}{29}$

$= \dfrac{22-32i}{29}$ or $\dfrac{22}{29}-\dfrac{32}{29}i$

SDWK

$i^2 = -1$

$(2i)^2 = 2^2 i^2$

$= 4(-1)$

$= -4$

#38. $i^{53} = i^{52}\cdot i^1$

$= (i^4)^{13}\cdot i$

$= (1)^{13}\cdot i$

$= 1\cdot i$

$= i$

#39 $i^{58} = i^{56}\cdot i^2$

$= (i^4)^{14}\cdot(-1)$

$= (1)^{14}\cdot(-1)$

$= 1\cdot(-1)$

$= -1$

SDWK

$4\overline{)53}$ → 13 R 1

-4

13

-12

1

$13 \times 4 = 52$

$4\overline{)58}$ → 14 R 2

-4

18

-16

2

$14 \times 4 = 56$

$i^2 = -1$

Math 101 Chapter 10 Review

#40. $\sqrt{132} \approx 11.48912529\ldots$
≈ 11.5

#41. $4\sqrt{43} \approx 4 \cdot (6.557438524\ldots)$
$\approx 26.22975411\ldots$
≈ 26.2

#42. $\sqrt[3]{31} \approx 3.141380652\ldots$
≈ 3.1

#44. $-10^{2/3} = -\sqrt[3]{10^2}$
$= -\sqrt[3]{100}$
$\approx -4.641588834\ldots$
≈ -4.6

#43. $\sqrt{11} + \sqrt[5]{5} \approx (3.31662479\ldots) + (1.379729661\ldots)$
$\approx 3.317 + 1.380$
≈ 4.697
≈ 4.7

#45. $18^{1/5} = \sqrt[5]{18}$
$\approx 1.782602458\ldots$
≈ 1.8

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#46. $(-17)^{2/3} = (\sqrt[3]{-17})^2$

$\approx (-2.571281591\ldots)^2$

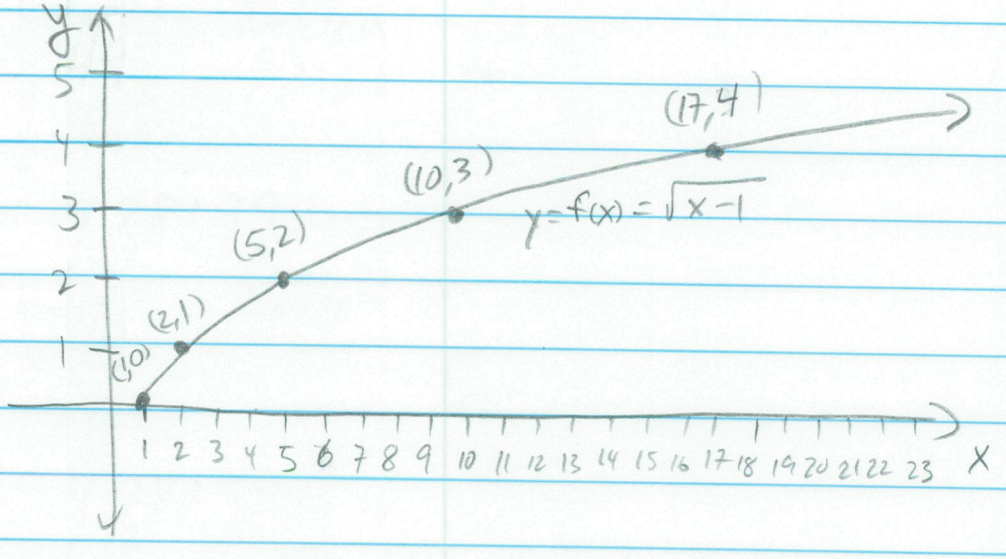
$\approx (-2.57)^2$

≈ 6.6049

≈ 6.6

#47. $f(x) = \sqrt{x-1}$

x	$f(x) = \sqrt{x-1}$
1	$\sqrt{1-1} = \sqrt{0} = 0$
2	$\sqrt{2-1} = \sqrt{1} = 1$
5	$\sqrt{5-1} = \sqrt{4} = 2$
10	$\sqrt{10-1} = \sqrt{9} = 3$
17	$\sqrt{17-1} = \sqrt{16} = 4$
26	$\sqrt{26-1} = \sqrt{25} = 5$



#48. $f(x) = \sqrt{x-1}$

$f(4) = \sqrt{(4)-1}$

$f(4) = \sqrt{3}$

$f(4) \approx 1.732050808\ldots$

$f(4) \approx 1.7$

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#49. $f(x) = \sqrt{x-1}$

find x when $f(x) = 7$

Solve:

$7 = \sqrt{x-1}$

$(7)^2 = (\sqrt{x-1})^2$

$49 = x - 1$

$49 + 1 = 1 + x - 1$

$50 = x$, $\{50\}$

check

$7 = \sqrt{(50)-1}$

$7 = \sqrt{49}$

$7 = 7$

TRUE!

#50. $f(x) = \sqrt{20x}$

x = length of skid marks (ft)

$f(x)$ = speed of a car (mph)

a. 255 ft skid mark = x

$f(255) = \sqrt{20(255)}$

$f(255) = \sqrt{5100}$

$f(255) = \sqrt{100} \cdot \sqrt{51}$

$f(255) = 10\sqrt{51}$

$f(255) \approx 10(7.141428542854\ldots)$

$f(255) \approx 10(7.1414)$

$f(255) \approx 71.414$

$f(255) \approx 71.4$

ANS: The car was traveling at approximately 71.4 miles per hour.

b. 60 mph = $f(x)$, find x

Solve:

$60 = \sqrt{20x}$

$(60)^2 = (\sqrt{20x})^2$

$3600 = 20x$

$\frac{3600}{20} = \frac{20x}{20}$

$180 = x$

ANS: Traveling at 60 mph, the car should skid approximately 180 ft before stopping.

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#51 $T(x) = \sqrt{\frac{x}{16}}$, $T(x)$ = time, in seconds, for an object to fall x feet

x = distance, in feet, an object falls

a. Find $T(x)$ when $x = 250$ feet

$$T(250) = \sqrt{\frac{(250)}{16}}$$

$$T(250) = \frac{\sqrt{250}}{\sqrt{16}}$$

$$T(250) = \frac{\sqrt{25}\sqrt{10}}{4}$$

$$T(250) = \frac{5\sqrt{10}}{4}$$

$$T(250) \approx (1.25)\cdot(3.162277601\ldots)$$

$$T(250) \approx (1.25)(3.162)$$

$$T(250) \approx 3.9525$$

$$T(250) \approx 4.0 \text{ seconds}$$

▷ ANS: It should take approximately 4 seconds for an object to fall 250 feet and hit the ground.

b. $T(x) = 3$ seconds, find x:

$$3 = \sqrt{\frac{x}{16}}$$

$$(3)^2 = \left(\sqrt{\frac{x}{16}}\right)^2$$

$$9 = \frac{x}{16}$$

$$16 \cdot 9 = \frac{16}{1} \cdot \frac{x}{16}$$

$$144 = x$$

▷ ANS: If an object falls for 3 seconds, it fell from a height of approximately 144 feet.